Post-Christmas IB NST Physics A Test

January 2024

You have one hour. Complete **all** questions. The number of marks available for each part of a question is shown in square brackets. In questions A1, A2, and A3, which are worth 5 marks each, answers should be brief, and relevant formulae may be assumed without proof. Question B4 is worth 20 marks.

A1

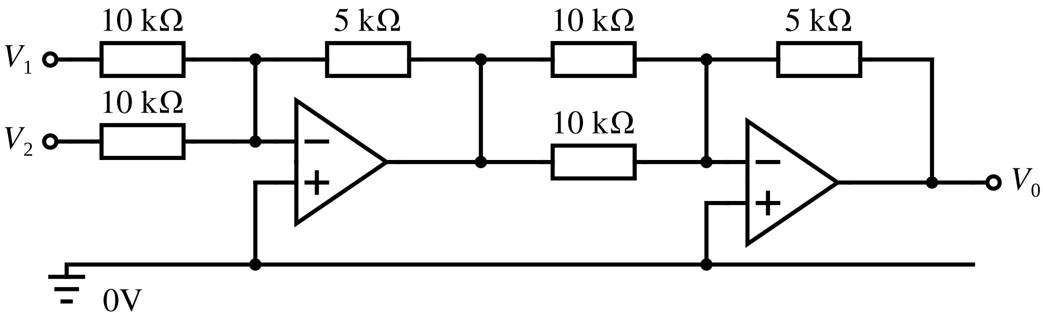
Oscar needs to manufacture twelve identical rhombi, each including an angle of $30°$, in order to make a decorative star. When he makes a rhombus, the actual angle he makes is $(30 \pm 1)°$. Define the "total angle" of the star as the sum of all twelve rhombus angles. Clearly it should be $360°$, and must be close to that if the star is to fit together well. But there is an error.

a) When making his first star, Oscar manufactures each of the twelve rhombi independently of the others. Find the error in the total angle. [2]

b) When making his second star Oscar manufactures the first rhombus as usual, then uses it as a template for the second rhombus. Each time he makes a copy in this way, he introduces an extra $\pm 1°$ of error, independently of the error in the thing copied. He then uses the second rhombus as a template for the third, the third as a template for the fourth, and so on. Find the error in the total angle for this second star. [3]

A2

Find a formula for V_0 in terms of V_1 and V_2 for the following circuit, assuming that neither op-amp is saturated. (The op-amps are both connected to an appropriate power supply.) [5]



A3

A hanging lamp may be modelled as a simple pendulum behaving as a lightly damped harmonic oscillator for small angles. The time period of the pendulum is 2.0s. The Q-value may be taken as 123. There is a point source of light of wavelength 550nm in the pendulum bob.
Two such hanging lamps, A and B, are hung very close together, with A due East of B. Lamp B is set swinging in a North-South plane with an amplitude of 25mm, and allowed to decay freely. The lamps are viewed from a distance 12m away, to the West of both lamps, using a telescope with an objective lens diameter of 25mm. Find the time taken for the oscillation to decay such that the images of lamps A and B can no longer be resolved from each other using the telescope, at any point in the swing, using the Rayleigh criterion. [5]

B4

A string under tension T transmits transverse waves of small amplitude. The mass per unit length of the string is μ.

a) Give the formula for the wavespeed v in terms of T and μ. [1]

b) Define the impedance, Z, of waves on this string. Give the formula for Z in terms of T and μ. [2]

c) The string is terminated by a massless ring free to slide on a frictionless rod. Consider an incident wave and reflected wave with transverse displacements $Ae^{i(kx-\omega t)}$ and $Are^{i(-kx-\omega t)}$ respectively, where k is the wavenumber, x the position along the string such that $x = 0$ at the ring, ω is the angular frequency, and t is time. By considering the boundary condition at $x = 0$, derive the reflexion coefficient r for this case, and describe the superposition of the incident and reflected waves. [3]

d) The ring is now attached to one end of a spring with spring constant K, where the axis of the spring lies along the rod. (The other end of the spring is fixed.) Derive the reflexion coefficient for this case. Write this reflexion coefficient in the form

$$\frac{Z_1 - Z_2}{Z_1 + Z_2} \quad ,$$

where Z_1 is the impedance of the string given in (b), and you should find Z_2 in terms of K and ω only. [5]

e) The ring is now replaced by a ring of mass $M \neq 0$. The spring is still connected as in part (d). Derive the reflexion coefficient for this case. Find (as in (d)) a formula for the new Z_2 in terms of K, M, and ω. [4]

f) Find the power reflexion coefficient corresponding to (e), and explain why this value is to be expected. [2]

g) For fixed M and K, find the value of ω that results in the same reflexion coefficients for (c) and (e), and explain why this value is to be expected. [2]

h) For fixed M, K, and Z_1 in the case of (e), find the limiting value of r as ω approaches infinity, and comment. [1]